Hard Momentum Questions   Name ……………………….

1. Thrust SSC is a supersonic car powered by 2 jet engines giving a total thrust of 180 kN.

   Calculate the impulse applied to the car when the engines run for 4 seconds.

   Assume the thrust is the only force acting on the car, which has a mass of 10 000 kg.

   Calculate the increase in speed of the car after the 4 s.

2. One suggestion for powering spacecraft engines of the future is an ion engine. A beam of ions (charged atoms) is fired backwards, propelling the spacecraft forwards. In one test, xenon ions were used.

   Consider how using xenon ions would compare with using krypton ions, which are lighter.

   If equal numbers of each ion were propelled back per second, at the same speed, which type of ion would you expect to give more thrust? Explain your answer

   The mass of a xenon ion is 2.2×10^{-25} kg, and it can be ejected at a speed of 3.1×10^{4} m s^{-1}.

   Calculate the number of ions that would have to be emitted per second to generate a thrust of 0.1 N (a typical value of the thrust from such an engine).

3. A spacecraft is approaching the planet Zog and needs to slow down. To do so it fires a jet of gas forwards.

   Explain how firing gas forwards slows the rocket down.

   The rocket has a mass of 50 000 kg. The gas can be fired forward at a speed of 5 000 m s^{-1} relative to the rocket. Calculate the mass of gas must the rocket eject to reduce its speed by 5 m s^{-1}. Ignore the change in the rocket's mass due to the ejection of gas.

4. 25 kg s^{-1} of air at 120 m s^{-1} is taken in by a jet engine that burns 1 kg fuel each second.

The exhaust gasses are ejected at 520 m s^{-1} relative to the engine.

a) Calculate the velocity change of the air.

b) Calculate the momentum change per second of the air and also of the fuel.

c) What is the thrust of the jet engine?

5. The H-3 Sea King Helicopter has a mass of about 5400 kg (including crew). It is hovering over the sea on a rescue mission. The rotors have a radius of about 10 m. Air density is 1.2 kg m^{-3}. Assume g = 10 N kg^{-1}

a) What lift force is needed to keep the helicopter hovering?

b) What downward velocity is given to the air by the hovering helicopter?

Practical Advice

These questions practise the analysis of momentum and impulse applied to jets and rockets. To make a valid attempt at all of them requires a high degree of familiarity with the basic terminology and equations. Apart from the first question they are definitely not for warm-up. The second question is based on a real technology. Question 5 is quite tough.

Answers and Worked Solutions

1. $$\text{Thrust} = Ft = 180000 \times 4 = 720000 \text{ N s}$$
$$Ft = mv - mu$$
$$720000 \text{ N s} = 10000 \times \Delta v$$
$$\Delta v = 72 \text{ m s}^{-1}$$

2. Xenon ions would provide more thrust. This is because there would be a greater momentum change per second since they have a greater mass than krypton ions. Let

m = mass of each ion, n = number of ions emitted per second, v = speed of ejection of ions.

Then, impulse, $F \Delta t$ = change in momentum = (m n) (v – 0): so

$$n = \frac{F\Delta t}{mv} = \frac{0.1 \text{ N} \times 1 \text{ s}}{(2.2 \times 10^{-25} \text{ kg}) \times (3.1 \times 10^{4} \text{ m s}^{-1})} = 1.5 \times 10^{19} \text{ s}^{-1}.$$

3. To eject the gas, the rocket exerts a forward force on the gas. By Newton's third law, the gas exerts a force of equal size back on the rocket. This force is responsible for the deceleration of the rocket.

$$\Delta p_{\text{gas}} = \Delta p_{\text{rocket}}$$
$$m_{\text{gas}} \Delta v_{\text{gas}} = m_{\text{rocket}} \Delta v_{\text{rocket}}$$
$$m_{\text{gas}} = \frac{50\,000 \text{ kg} \times 5 \text{ m s}^{-1}}{5000 \text{ m s}^{-1}} = 50 \text{ kg}.$$

4. a) Velocity change of the air = (520 -120) = 400 m s^{-1}

b) Momentum change per second of the air = 25 x 400 = 10,000 kg m s^{-2}

Momentum change per second of the fuel = 1 x 520 = 520 kg m s^{-2}

c) Engine thrust $F = \frac{m\Delta v}{\Delta t}$ = total momentum change/second = 10,000 + 520 =

10,520 N

5. a) Force is needed to keep the helicopter hovering = weight = 5400 x 10 = 54000 N

b) $$\text{downward force on air } F = \frac{\Delta p}{\Delta t} = \frac{v\Delta m}{\Delta t}$$
$$\text{but } \frac{\Delta m}{\Delta t} = \rho A v$$
$$\text{so } F = \rho A v^2$$
$$\text{downward force on air} = \text{weight of helicopter}$$
$$\rho A v^2 = m_b g$$
$$v = \sqrt{\frac{m_b g}{\rho A}}$$
$$v = \sqrt{\frac{54000}{1.2 \text{ kg m}^{-3} \times (\Pi x 10 x 10) \text{ m}^2)}}$$
$$v = 12 \text{ m s}^{-1}$$